REAL-TIME DYNAMICS OF
PARTON-HADRON CONVERSION

J. Ellis and K. Geiger

CERN TH-Division, CH-1211 Geneva 23, Switzerland

Abstract

We propose a new and universal approach to the hadronization problem that incorporates both perturbative QCD and effective field theory in their respective domains of validity and that models the transition between them in analogy to the finite temperature QCD phase transition. Using techniques of quantum kinetic theory, we formulate a real-time description in momentum and position space. The approach is applied to the evolution of fragmenting $q\bar{q}$ and gg jets as the system evolves from the initial 2-jet, via parton multiplication and cluster formation, to the final yield of hadrons. We investigate time scale of the transition, energy dependence, cluster size and mass distributions, and compare our results for particle production and Bose-Einstein correlations with experimental data for $e^+e^- \rightarrow hadrons$. An interesting possibility to extract the space-time evolution of the system from Bose enhancement measurements is suggested.

johne@cernvm.cern.ch

klaus@surya11.cern.ch

0

The physics of QCD is well understood in two limits: at short distances ($\ll 1$ fm) where the relevant degrees of freedom are quarks and gluons, the dynamics of which is accurately described by perturbative QCD, and at large distances ($\gtrsim 1$ fm) where the relevant degrees of freedom are hadrons whose non-perturbative interactions are well described by chiral models. What is less understood, and constitutes one of the key open problems in QCD, is the transition between the short- and long-distance regimes through intermediate distance scales. This problem is particularly serious for attempts to describe the phenomenon of hadronization in high energy QCD processes. Perturbative QCD describes quantitatively the short-distance evolution of the dynamical system and hence the large-scale flow of energy-momentum. The transformation of colored quarks and gluons into colorless hadrons, however, is commonly believed not to involve large momentum transfers, as expected for a large-distance effect. Moreover, experiments on many high-energy QCD processes, including for example $e^+e^- \rightarrow hadrons$ and deep inelastic lepton-nucleon scattering, strongly support the idea that the fragmentation of partons into hadrons is a universal mechanism. However, the theoretical tools currently available for studying QCD are inadequate to describe the dynamics of this transformation from partonic to hadronic degrees of freedom. Perturbative techniques are limited to the short distance regime where confinement is not apparent [1], whilst effective low-energy chiral models [2] and QCD sum rules [3] that incorporate confinement, lack partonic degrees of freedom. In principle, lattice QCD [4] should be able to bridge the gap, but in practice dynamical calculations of parton-hadron conversion are not yet feasable.

In this paper we extend previous work [5] and advocate a new approach to the hadronization problem that incorporates both perturbative QCD and effective field theory in their respective domains of validity and that models the transition between them using ideas developed in phenomenological descriptions of the finite temperature transitions from a quark-gluon plasma to a hadronic phase [6]. The latter is described by an effective theory that incorporates a chiral field U whose vacuum expectation value (vev) $U_0 \equiv \langle 0|U + U^\dagger|0\rangle$ represents the quark condensate $\langle 0|\bar{q}q|0\rangle$, and a scalar field χ whose vev $\chi_0 \equiv \langle 0|\chi|0\rangle$ represents the gluon condensate $\langle 0|F_{\mu\nu}F^{\mu\nu}|0\rangle$. We visualize a high energy collision such as $e^+e^- \rightarrow \bar{q}q$ as producing a "hot spot" in which the long range order, represented by U_0 and χ_0, is disrupted locally by the appearance of a bubble of the naive perturbative vacuum in

which $\langle 0|\bar{q}q|0\rangle = 0 = \langle 0|F_{\mu\nu}F^{\mu\nu}|0\rangle$. Within this bubble, a parton shower develops in the usual perturbative way, with the hot spot expanding and cooling in an irregular stochastic manner described by QCD transport equations. This perturbative description remains appropriate in any phase-space region of the shower where the local energy density is large compared with the difference in energy density between the perturbative partonic and the non-perturbative hadronic vacua. When this condition is no longer satisfied, a bubble of hadronic vacuum may be formed with a probability determined by statistical-mechanical considerations. A complete description of this conversion requires a treatment combining partonic and hadronic degress of freedom, which is the essential aspect of our approach. Although studies of the finite temperature QCD phase transition [7] indicate that it may be completed quite rapidly, in which case mixed description is not needed in a first approximate treatment, we are more ambitious here and propose a universal approach to the dynamic transition *between* partons and hadrons based on an effective QCD field theory description and relativistic kinetic theory that includes a mixture of both sets of degrees of freedom.

The purpose of this letter is to present the essential concepts of our approach, i.e. the quantitative formulation and its application of the above picture. An extensive documentation of our work can be found in Ref. [8] to which we refer for details. Let us begin by defining the *distance measure* L for the space-time separation between two points r and r' $[r \equiv r^\mu = (t, \vec{r})]$:

$$L := \sqrt{(r - r')_\mu (r - r')^\mu} \,, \tag{1}$$

and introduce a *characteristic length scale* L_c that separates short distance ($L \ll L_c$) and long range ($L \gtrsim L_c$) physics in QCD. The scale L_c can be associated with the confinement length of the order of a hadron radius. In the limit of short distances $L \ll L_c$ (or high momenta, high temperatures), QCD may be described perturbatively by a Lagrangian in terms of the *elementary* gluon (A^μ) and quark fields ($\psi, \overline{\psi}$),

$$\mathcal{L}_L[A^\mu, \psi, \overline{\psi}] \;=\; -\frac{\kappa_L}{4} \, F_{\mu\nu,a} F_a^{\mu\nu} \;+\; \overline{\psi}_i \left[\left(i\gamma_\mu \partial^\mu - \mu_L \right) \delta_{ij} - g_s \gamma_\mu A_a^\mu T_a^{ij} \right] \psi_j \,, \tag{2}$$

where $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_s f_{abc} A_b^\mu A_c^\nu$ and summation over color indices a, i, j is understood. The functions κ_L and μ_L introduce an explicit scale(L)-dependence in $\mathcal{L}_L$ which modifies the quark and gluon properties when L increases towards L_c and beyond. In

the limit $L \to 0$, $\kappa_L = 1$ and $\mu_L = 0$ (neglecting the quark current masses), thus the QCD Lagrangian is recovered. However, at larger L, the bare quark and gluon fields become dressed by non-perturbative dynamics and we expect $\kappa_L < 1$ and $\mu_L \neq 0$. In fact, the short range behaviour of κ_L (and similarly of μ_L) can be calculated perturbatively: $\kappa_L = [1 + g_s^2/(8\pi)^2(11 - 2n_f/3)\ln(L\,\Lambda)]^{-1}$. Hence both κ_L and the dynamical quark masses μ_L vary (as does the coupling constant g_s) with the renormalization scale Λ, which we expect to be directly related to the confinement scale L_c. We will return to that issue below.

In the limit of large distances $L \gtrsim L_c$ (or low momenta, low temperatures) the hadronic physics can well be described by an effective field theory that embodies the scale and chiral constraints of the fundamental QCD Lagrangian. The corresponding effective Lagrangian is written in terms of *collective* fields χ, $U, U^\dagger$ [6]:

$$\mathcal{L}[\chi, U, U^\dagger] = \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) + \frac{1}{4}Tr\left[(\partial_\mu U)(\partial^\mu U^\dagger)\right] - V(\chi, U), \tag{3}$$

where χ is the aforementioned scalar field and $U = f_\pi \exp\left(i\sum_{j=0}^8 \lambda_j \phi_j/f_\pi\right)$ the pseudoscalar field for the nonet of meson fields ϕ_j, and the potential $V(\chi, U)$ is given by

$$
\begin{aligned}
V(\chi, U) &= b\left[\frac{1}{4}\chi_0^4 + \chi^4 \ln\left(\frac{\chi}{e^{1/4}\chi_0}\right)\right] + \frac{1}{4}\left[1 - \left(\frac{\chi}{\chi_0}\right)^2\right]Tr\left[(\partial_\mu U)(\partial^\mu U^\dagger)\right] \\
&\quad + c\,Tr\left[\hat{m}_q(U + U^\dagger)\right]\left(\frac{\chi}{\chi_0}\right)^3,
\end{aligned}
\tag{4}
$$

which models the quantum dynamics in the low energy regime associated with the self- and mutual interactions of χ and U. Here the parameter b is related to the conventional bag constant B by $B = b\chi_0^4/4$, c is a constant of mass dimension 3, $m_q = \mathrm{diag}(m_u, m_d, m_s)$ is the light quark mass matrix. The *vev*'s of the collective fields χ and U are zero in the short distance limit $L \ll L_c$, i.e. in the naive perturbative vacuum, but become finite in the long wavelength limit at large distances $L \gtrsim L_c$, namely $\langle 0|\chi|0\rangle = \chi_0$ and $\langle 0|U + U^\dagger|0\rangle = U_0$, which are to be regarded as order parameters of the physical vacuum. Thus, the potential (4) has a minimum when $\langle \chi \rangle = \chi_0$ and equals the vacuum pressure (bag constant) B at $\langle \chi \rangle = 0$.

As already advertised, we seek to describe parton-hadron conversion in the aftermath of a high-energy collision, in which an initial "hot spot" expands irregularly. Inhomogenities that appear during the expansion are characterized by local dynamical scales. It is well

known how to incorporate small distance (high momentum) scales using the renormalization group in perturbative QCD. Dynamical scale dependence can also be taken into account in the long distance effective Lagrangian (3). For instance, at finite temperature T the effective potential (4) acquires additional T-dependent terms [6], resulting in a modification of the vev's of χ and $U + U^\dagger$ and thus of the gluon and quark condensdates, $\langle 0|F_{\mu\nu}F^{\mu\nu}|0\rangle$ and $\langle 0|\bar{q}q|0\rangle$. In the present situation we are dealing however with general non-equilibrium systems, in which we advocate the length scale L to control the scale dependence of local fluctuations. A complete treatment of the hadronization process clearly requires the linkage between the elementary partonic degrees of freedom (A^μ, ψ) and the collective hadronic degrees of freedom (χ, U). From the above discussion we expect that

$$\lim_{L\to 0} \kappa_L = 1 \,, \qquad \lim_{L\to 0} \mu_L = 0 \,, \qquad \lim_{L\to 0} \langle\chi\rangle_L = 0$$
$$\lim_{L\to\infty} \kappa_L = 0 \,, \qquad \lim_{L\to\infty} \mu_L = \infty \,, \qquad \lim_{L\to\infty} \langle\chi\rangle_L = \chi_0 \tag{5}$$

corresponding to the unconfined and confined phases, respectively. In view of the one-to-one relation between L and $\langle\chi\rangle_L$ one may express (5) as boundary conditions of the $\langle\chi\rangle$-dependence of κ_L and μ_L when $\langle\chi\rangle \to 0$ or $\langle\chi\rangle \to \chi_0$. This characterizes the so-called color-dielectric property of the vacuum in QCD [9], which has been widely applied in soliton models and hadron phenomenology.

It is important to realize that the variation of the internal length scale L, as defined in (1), is governed by the dynamics of the fields itself and in turn it must determine the time evolution of the fields A^μ, ψ as well as χ. With regard to (5), we assume that the behavior of the functions κ_L and μ_L is correlated with the value of $\langle\chi\rangle$ and express this by $\kappa_L \equiv \kappa_L(\chi)$ and $\mu_L \equiv \mu_L(\chi)$. Then, by combining (2)-(4), we obtain an effective field theory description covering the full range $0 < L < \infty$,

$$\mathcal{L}_L[A^\mu, \psi, \chi, U] := \mathcal{L}_L[A^\mu, \psi, \overline{\psi}] + \mathcal{L}[\chi, U, U^\dagger] \,, \tag{6}$$

in which the elementary gluon and quark fields are coupled to the collective field χ via $\kappa_L(\chi)$ and $\mu_L(\chi)$ in a self-consistent manner that is controlled by the dynamically varying scale L. Our present understanding of QCD does not yield us explicit interpolating formulae for κ_L and μ_L from first principles. Fortunately, as we find, the particularities of their functional forms are not crucial for the treatment of parton hadron conversion that we present here,

as long as they are smooth and satisfy the boundary conditions (5). The reason is that, as L decreases, the appropriate dynamical description of QCD apparently jumps rapidly from the first limit in (5) to the neighborhood of the second limit, corresponding to the rapid formation of hadronic domains when $L \gtrsim L_c$. This feature is very reminiscent of the weakly first order nature of the QCD phase transition at finite temperature. The essential point is that, due to the conditions (5), the variation of κ_L with L and χ generates color charge confinement at large distances, because of the fact that a color electric charge creates a displacement $\vec{D}_a = \kappa_L \vec{E}_a$, where $E_a^k = F_a^{0k}$, with energy $\frac{1}{2} \int d^3 r D_a^2 / \kappa_L$ which becomes infinite at large L for non-zero total charge. Similarly, the increase of the dynamical mass μ_L with L and χ leads to an effective confinement potential that ensures absolute confinement also for quarks by prohibiting their propagation at large distances. We tested various choices for κ_L and μ_L and, to be specific, in what follows we adopt the forms $\kappa_L(\chi) = 1 - (L\chi)^2/(L_0\chi_0)^2$, which is a minimal possibility compatible with the QCD scaling properties, and $\mu_L(\chi) = \mu_0(\kappa_L^{-1}(\chi) - 1)$ with constant μ_0 set equal to 1 GeV. The latter form reflects that the quark mass term μ_L is assumed to be induced by non-perturbative gluon interactions (through κ_L), rather than being an independent quantity, as is suggested by an explicit calculation [10] of the quark self-energy involving the gluon propagator in the presence of the collective field χ.

Let us emphasize once more the similarity of the above approach with finite temperature QCD phenomenology [6]. The effect of κ_L and μ_L can be interpreted as a scale (L) dependent modification δV, which adds to the (L independent) potential V, eq. (4),

$$\mathcal{V}(L) := V(\chi, U) + \delta V(L, \chi) \ , \quad \delta V(L, \chi) = \frac{\kappa_L(\chi)}{4} F_{\mu\nu,a} F_a^{\mu\nu} + \mu_L(\chi) \overline{\psi}_i \psi_i \ , \quad (7)$$

with $\delta V = O(L^2)$. Here the length scale L corresponds to the temperature T in finite temperature QCD, where the correction to the zero temperature potential is $O(T^2)$. Hence we expect the characteristic confinement length L_c to play a similar role as the critical temperature T_c in the QCD phase transition. However, this formal anology is to be taken with some caution, because here we are aiming to describe the evolution of a general non-equilibrium system in real time and Minkowski space, as opposed to the thermal evolution in Euclidian space.

We complete our discussion of the above field theory aspects with the following important remarks: (i) The formulation is gauge and Lorentz invariant and is consistent with

scale and chiral symmetry properties of QCD. It interpolates from the high momentum (short distance) QCD phase with unconfined gluon and quark degrees of freedom and chiral symmetry ($\langle\chi\rangle = 0, \langle U\rangle = 0, \kappa_L = 1, \mu_L = 0$), to a low energy (long range) QCD phase with confinement and broken chiral symmetry ($\langle\chi\rangle = \chi_0, \langle U\rangle = U_0, \kappa_L = 0, \mu_L = \infty$). (ii) By construction (6) strictly avoids double counting of degrees of freedom, because the introduction of the scale L and the behaviour of the L-dependent the functions κ_L and μ_L truncate the dynamics of the elementary fields A^μ, ψ to the short distance regime, whereas the effective description in terms of the collective fields χ, U covers the complementary long range domain. (iii) There is no need for explicit renormalization of the composite fields χ and U, because they are already interpreted as effective long range degrees of freedom with loop corrections implicitly included in $V(\chi, U)$ and it would be double counting to add them again.

As documented in detail in [8, 11], it is possible to derive from the Lagrangian (6) a formulation of QCD transport theory that incorporates both partons and hadrons, yielding a fully dynamical description of the QCD matter in real time and complete phase-space. Since this serves as the basis for our subsequent description of the conversion of partons into hadrons locally in phase-space, we sketch here this formulation of QCD transport theory. The first step is to derive from the field equations of motion the corresponding Dyson-Schwinger equations for the *real-time Greens fuctions* of the fields ψ, A^μ, χ, and U, denoted by $G_\alpha(x, y)$, and where $\alpha \equiv q, g, \chi, U$. They are defined as the two-point functions that measure the time ordered correlations between the fields at space-time points x and y. In symbolic notation, one obtains a coupled system of integral equations,

$$G_\alpha(x, y) \;=\; G_\alpha^{(0)}(x, y) \;+\; \sum_\beta \int d^4x' d^4x'' \, G_\alpha^{(0)}(x, x') \, \Sigma_\beta(x', x'') \, G_\alpha(x'', y) \,, \qquad (8)$$

where $G_\alpha^{(0)}$ denotes the free field Greens functions that satisfy the equations of motion in absence of interactions, and the *self-energies* Σ_α embody both the mutual and the self-interactions of the fields. The explicit expressions can be found in Ref. [11]. Next we introduce the Wigner transforms

$$\mathcal{G}_\alpha(r, p) \;=\; \int d^4R \, e^{ip \cdot R} \, G_\alpha(x, y) \,, \qquad (9)$$

(where $r = \frac{1}{2}(x + y)$, and p is the conjugate variable to $R = x - y$) of the Greens functions and similarly for the self-energies $\Sigma_\alpha(x, y)$. The dependences on r would be trivial for

6

field theory *in vacuo*, but non-trivial in an inhomogenous QCD matter. In terms of the Wigner transforms $\mathcal{G}_\alpha(r,p)$ and $\tilde{\Sigma}_\alpha(r,p)$, the Dyson-Schwinger equations (8) become kinetic equations describing the real-time evolution of the matter in phase-space spanned by $\vec{r}$ and $p^\mu = (E, \vec{p})$. Then, by tracing over color and spin polarizations, and taking the expectation values (or, in medium the ensemble average) of the Wigner transformed Greens functions $\mathcal{G}_\alpha$, one obtains the scalar functions

$$F_\alpha(r,p) \;\equiv\; F_\alpha(t,\vec{r};\vec{p}, p^2 = M_\alpha^2) \;=\; \langle\, Tr[\mathcal{G}_\alpha(r,p)]\,\rangle\,\Big|_{M_\alpha^2 = \tilde{\Sigma}_\alpha(r,p)}. \tag{10}$$

The *c*-number functions $F_\alpha(r,p)$ for the particle species α are the quantum mechanical analogues to the classical phase-space distributions that measure the number of particles at time t in a phase-space element $d^3r\,d^4p$. The F_α contain the essential microscopic information required for a statistical description of the time-evolution of a many-particle system in complete phase-space and provide the basis for calculating macroscopic observables in the framework of relativistic kinetic theory. In particular, the local space-time dependent particle currents n_α for the different particle species and the corresponding energy-momentum tensors $T_\alpha^{\mu\nu}$ are given by [12]

$$n_\alpha^\mu(r) \;=\; \int d\Omega_\alpha\, p^\mu F_\alpha(p,r)\,, \qquad T_\alpha^{\mu\nu}(r) \;=\; \int d\Omega_\alpha p^\mu p^\nu\, F_\alpha(r,p)\,, \tag{11}$$

where $d\Omega_\alpha = \gamma_\alpha dM^2 d^3p/(16\pi^3 p^0)$, the γ_α are degeneracy factors for the internal degrees of freedom (color, spin, etc.), M_α measures the amount by which a particle α is off mass-shell as a result of the selfenergy terms in (8), and $p^0 \equiv E = +\sqrt{\vec{p}^2 + M_\alpha^2}$. These macroscopic quantities can be written in Lorentz invariant form by introducing for each species α the associated matter flow velocity $u_\alpha^\mu(r)$, defined as a unit-norm time-like vector at each space-time point, $(u_\mu u^\mu)_\alpha = 1$. A natural choice is e.g. $u_\alpha^\mu = n_\alpha^\mu/\sqrt{n_{\nu\,\alpha} n_\alpha^\nu}$. By contracting the quantities (11) with the local flow velocities u_α^μ, one can now obtain corresponding invariant scalars of particle density, pressure, and energy density, for each particle species α individually,

$$n_\alpha(r) \;=\; n_{\mu\,\alpha}\, u_\alpha^\mu\,, \quad P_\alpha(r) \;=\; -\frac{1}{3} T_{\mu\nu,\,\alpha}\,(g^{\mu\nu} - u_\alpha^\mu u_\alpha^\nu)\,, \quad \varepsilon(r) \;=\; T_{\mu\nu,\,\alpha}\, u_\alpha^\mu u_\alpha^\nu\,. \tag{12}$$

The above formulation is readily applicable to the dynamics of the parton-hadron conversion in rather general situations. Here we will as an illustrative application study the fragmentation of a $q\bar{q}$ jet system with its emitted bremsstrahlung gluons and describe the

7

evolution of the system as it converts from the parton phase to the hadronic phase. In this case the kinetic equations that derive from the Dyson-Schwinger equations (8) simplify considerably and yield a set of coupled tranport equations for the phase-space densities $F_\alpha(t, \vec{r}; \vec{p}, M_\alpha^2)$ of the generic form [8]

$$\left[p_\mu \, \partial_r^\mu \; + \; (\overline{M}_\alpha \, \partial_r^\mu \overline{M}_\alpha) \, \partial_\mu^p \right] \, F_\alpha \;\; = \;\; \sum_{processes \; k} \left[\hat{I}_k^{(+)}(F_\beta) \; - \; \hat{I}_k^{(-)}(F_\beta) \right] \, F_\alpha \, . \qquad (13)$$

These kinetic equations reflect a probabilistic interpretation of the evolution in terms of successive interaction processes k, in which the change of the particle distributions F_α is governed by the balance of gain $(+)$ and loss $(-)$ terms. The left hand left side describes propagation of a quantum of species α in the presence of the mean field $\overline{M}_\alpha$ generated by the others, and on the right hand side the integral operators $\hat{I}^{(\pm)}$ incorporate the effects of the self-energies in terms of real and virtual interactions that lead to particle excitations.

We now discuss the illustrative application of the outlined formalism to parton-hadron conversion starting from a highly virtual γ^* or a Z^0 in an $e^+ e^-$ annihilation event with large invariant mass $Q^2 \gg \Lambda^2$ corresponding to a very small initial $L \ll 1 \; fm$, that initiates a $q\bar{q}$ jet evolution. At first the partons multiply, determined by a coherent parton shower simulation incorporating (coherent) gluon emission, secondary $q\bar{q}$ production, etc.. Eventually, as the quanta diffuse in space-time and L increases, they will coalesce to pre-hadronic clusters by tunneling through the potential barrier imposed by the dynamically changing $\mathcal{V}(L)$, which is determined within a coalescence model. Finally these pre-hadronic clusters will convert into physical hadronic states and subsequently decay into low mass hadrons, depending on the density of accessible hadronic states. The system of particles is evolved in discrete time steps, here taken as $\Delta t = 0.01 \; fm$, in coarse grained 7-dimensional phase-space with cells $\Delta \Omega = \Delta^3 r \Delta^3 p \Delta M^2$. The partons propagate along classical trajectories until they interact, i.e. decay (branchings) or recombine (cluster formation). Similarly, the formed clusters travel along straight lines until they decay into hadrons. The corresponding probabilities and time scales of interactions are sampled stochastically from the relevant probability distributions. With this concept, we can trace the space-time evolution of the system [8]: In each time step, any "hot" off-shell parton is allowed to decay into "cooler" partons, with a probability determined by its virtuality and life-time. Also in each step, every parton and its nearest spatial neighbor are considered as defining a fictious pre-hadronic

8

bubble with invariant radius L, as defined by (1). Depending on the value of L, the relative probabilities for which configuration is more favorable determine whether the partons continue in their shower development, or a parton cluster is formed. This cascade evolution is followed until all partons have converted, and all clusters have decayed into final hadrons.

To this end we need to specify the parameters in the potential V, eq. (4), or $\mathcal{V}(L)$, eq. (7). As L varies, $\mathcal{V}$ changes its shape and affects the evolution of the system. The details of the dynamics are controlled the choice of bag constant B which defines the vacuum pressure $V(0)$ in the short distance limit $L \to 0$, and χ_0, the value of the condensation of χ in the long distance regime. Although the values of B and χ_0 are not precisely known, there is agreement of various phenomenological determinations about their range: one expects [6] $B^{1/4} = (150 - 250)$ MeV and $\chi_0 = (50 - 200)$ MeV. In what follows, we choose two representative combinations: $(B^{1/4}, \chi_0) = (230, 200)$ MeV and $(B^{1/4}, \chi_0) = (180, 100)$ MeV. We then proceed in analogy to the QCD phase transition model of Ref. [6] and compare in each time step the local pressure of partons $P_{qg}(t, \vec{r}, L)$ with the pressure $P_\chi(t, \vec{r}, L)$ that a pre-hadronic bubble would create instead. We compute the pressures from the corresponding phase-space densities (10) and the formulae (11),(12). Representing

$$ P_{qg}(r, L) \;=\; a_{qg}(r, L) \, L^{-4} \;-\; B \;, \quad P_\chi(r, L) \;=\; a_\chi(r, L) \, L^{-4} \;-\; \mathcal{V}(L) \;, \quad (14) $$

and, defining $L = L_c$ as the characteristic length scale, in analogy to the critical temperature in the QCD phase transition, such that the two pressures equal each other at L_c and $\mathcal{V}(\chi, L_c) = 0$, we get the determining condition:

$$ L_c \;=\; \left[\frac{a_{qg}(r, L_c) \;-\; a_\chi(r, L_c)}{B} \right]^{1/4} \;. \quad (15) $$

With the functions a_{qg} and a_χ obtained from the numerical simulation, we find then for the characteristic confinement length $L_c = 0.6 \; fm$ for the choice $B^{1/4} = 230$ MeV and $L_c = 0.8$ fm for $B^{1/4} = 180$ MeV.

Fig. 1a compares the parton and hadron pressures $P(t, L)$ (calculated at the peak of the expanding shock front) as a function of time t in parton showers initiated by $q\bar{q}$-pairs with center-of-mass energies $Q = 100$ GeV. We see that the pressures cross over at a time $t \simeq 0.7 \; fm$, almost independent of the choice of B (and hence L_c), and we have found the same feature in gg-initiated showers. The corresponding time development of the total generated transverse momentum $p_\perp(t)$ is shown in Fig. 1b, where we see that the share

9

of hadronic clusters dominates when $t \gtrsim 1 \; fm$, which again is also true for gg-initiated showers. At lower energy (we performed an identical analysis at $Q = 10$ GeV) we found the same features, except that the crossover times are somewhat shorter.

In this dynamically evolving situation the probability that two partons with invariant spatial separation L coalesce to a cluster (viewed as a pre-hadronic bubble formed in the vacuum), is determined by $\pi(L) = 1 - \exp(-\Delta F \, L)$, where ΔF is the free energy of a thin-walled bubble of one phase (partons), immersed in a medium in the other phase (vacuum):

$$\Delta F \;=\; \frac{4\pi}{3} \, R_c^2 \, \sigma \;, \tag{16}$$

where $R_c = 2\sigma/\Delta P$ is the critical bubble size determined in terms of the surface tension σ and the difference in pressures $\Delta P = P_{qg} - P_\chi$. We estimate the surface tension from the relation [13] $\sigma \equiv \sigma(L_c) = \int d\chi \sqrt{2\mathcal{V}(L)}|_{L=L_c}$ with the potential (7), which yields $\sigma^{1/3} = 40$ (48) MeV (for $B^{1/4} = 230$ (180) MeV). We remark that similar numbers are indicated by lattice QCD simulations. These small values of σ correspond to a weakly first-order transition at finite temperature, which is consistent with astrophysical constraints on inhomogenities. As a consequence of this feature, although our approach allows for parton-hadron conversion when the parton separation L is even less than L_c, or, when $L \gg L_c$, most clusters in fact form when L is only slightly larger than L_c, as seen in Fig. 2a. We also observe that the distribution of cluster sizes is essentially independent of the initial state energy. The same feature was found true for gg-initiated showers. The spectrum of cluster masses, determined by the total invariant masses of coalescing partons at the moment of conversion, is shown in Fig. 2b which exhibits no sensitivity to the value of L_c at all. Moreover, the shapes of both spectra turn out to be rather independent of the jet energy, as well as of the type of initial partons, and therefore appear to be universal.

To obtain the hadron spectrum from the cluster distribution, we implement essentially the cluster-fragmentation scheme of [14], but with the important modification that a cluster weighing more than a critical mass $M_{crit} = 4 GeV$ is not allowed to convert from partons to hadrons, but is forced to continue its partonic shower development. This restriction is motivated by the experimental indication that very heavy cluster production is suppressed. Although conceptionally significant, it is not very important numerically, because it affects less than 5 % of the clusters. We show in Fig. 3a the total charged particle multiplicity n_{ch}, which grows with energy in consistency with experimental data. The momentum spectra

10

of charged hadrons with respect to $x = 2E/Q$, the particle energy normalized to the total energy Q, at $Q = 34$ GeV and $Q = 91$ GeV, are shown in Fig. 3b and again agree with the data. Parton shower Monte Carlos are well known to describe these data sucessfully: our repetition of this success simply means that our novel parton-hadron conversion mechanism is innocuous in this regard. The comparisons in Fig. 3 do not indicate any clear preference for one value of B or L_c over the other.

However, some indication may eventually be drawn from Bose-Einstein correlations among produced hadrons. The observed Bose enhancements in the pion spectra are generally interpreted as reflecting the size and coherence of the hadron emission region, which is directly related to the choice of B and thus L_c in our model. We have used the method of Sjöstrand [18] to simulate the enhancement for same-sign charged pion pairs. We see in Fig. 4a that the enhancements are indeed distinguishable for $B^{1/4} = 230$ MeV ($L_c = 0.6$ fm) and $B^{1/4} = 180$ MeV ($L_c = 0.8$ fm). Moreover, these enhancements are essentially independent of Q, as can be seen from the ratios of the enhancement factors plotted in Fig. 4b. We compare our predictions with OPAL data in Fig. 4a. *Prima facie*, these are very consistent with our predictions for $B^{1/4} = 180$ MeV ($L_c = 0.8$ fm). However, the unambiguous definition of a preferred value of this input parameter must await a systematic analysis of available data on Bose-Einstein correlations in $e^+e^- \rightarrow hadrons$. Nevertheless, we are pleased that our model appears to be able to correlate sucessfully such diverse quantities as the bag constant B, the characteristic microscopic scale L_c for the parton-hadron transition, and measured Bose-Einstein correlations.

In conclusion: we presented a novel approach to the dynamics of parton-hadron conversion and confinement, based on a kinetic multi-particle description in real time and complete phase-space, which combines perturbative QCD at high energies and an effective field theory approach to hadrons at low energies. As a test application that exhibits generic features, we have considered the prototype reaction $e^+e^- \rightarrow hadrons$. The main results are: (i) the local conversion of partons to hadronic clusters occurs very rapidly, but the global time scale for the transition of all parts of the system is comparatively long; (ii) features of the perturbative parton evolution are projected unscathed onto hadron distributions; (iii) the Bose enhancement of same-sign pions may provide a sensitive probe of the details of the space-time evolution. Our approach may be extended to other applications

involving parton cascades and hadronization in different environments, including deep inelastic lepton-nucleon or -nucleus scattering, high energy hadron-hadron, hadron-nucleus, or nucleus-nucleus collisions, and the (non-equilibrium) dynamics of the QCD phase transition at finite density and temperature. The development of a systematic understanding of the hadron emission regions in different processes appears within reach.

References

[1] Yu. L. Dokshitzer, D. I. Dyakonov, and S. I. Troyan, Phys. Rep. **58**, 269 (1980).

[2] E. Witten, Nucl. Phys. **B223**, 422, 433 (1983); J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984); U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991).

[3] M. A. Shifman, A. I. Vainshtein, and V. A. Zakharov, Nucl. Phys. **B147**, 448 (1979).

[4] F. Karsch, Nucl. Phys. (Proc. Suppl.) **B9**, 357 (1989).

[5] K. Geiger, Phys. Rev. **D51**, 2345 (1995).

[6] B. A. Campbell, J. Ellis, and K. A. Olive, Phys. Lett. **B235**, 325 (1990); Nucl. Phys. **B345**, 57 (1990).

[7] K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, CERN-TH.7220/94.

[8] J. Ellis and K. Geiger, preprint CERN-TH. 95-35.

[9] See e.g.: T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic Press, New York, 1981).

[10] G. Fai, J. Perry and L. Wilets, Phys. Lett. **B208**, 1 (1988); G. Krein, P. Tang, L. Wilets and A.G. Williams, Nucl. Phys. **A523**, 548 (1991).

[11] K. Geiger, in preparation.

[12] K. Geiger, Phys. Rep. in press, CERN-TH. 7313/94.

[13] S. Coleman, Nucl. Phys. **262**, 263 (1985).

[14] K. Geiger, Phys. Rev. **D47**, 133 (1993).

[15] S. Haywood, Rutherford-Appleton preprint RAL-94-074, and references therein.

[16] S. L. Wu, Phys. Rep. **107** (1984), 59; T. Hebbeker, Phys. Rep. **217** (1992), 69.

[17] Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller, and S. I. Troyan, Advanced Series on Directions in High Energy Physics, Vol. 5 (Editions Frontieres 1991)

[18] T. Sjöstrand, publication CERN-TH.7112/93.

FIGURE CAPTIONS

Figure 1: a) Time evolution of the kinetic pressures P_{qg} of partons and P_χ of prehadronic clusters for $q\bar{q}$ initiated jet evolution with total jet energy $Q = 100$ GeV. **b)** Total transverse momentum $p_\perp$ generated during the time evolution of the system in the center-of-mass of the initial two-jet.

Figure 2: a) Distribution of the cluster sizes of clusters formed from neighboring partons. **b)** Associated cluster mass spectrum.

Figure 3: a) Resulting average charged multiplicity versus total energy Q in e^+e^- annihilation events, in comparison with experimental data. **b)** x-spectra of charged hadrons $(x = 2E/Q)$ with respect to the variable $\ln(1/x)$ at $Q = 34$ GeV and $Q = 91$ GeV, confronted with measured disributions at PEP and LEP.

Figure 4: a) Simulated Bose-Einstein enhancement $b_{L_c}(q)$ as a function of pair mass q of emitted same-sign pion pairs for the two values of L_c. **b)** Ratios of the enhancements $b_{0.6\,fm}(q)/b_{0.8\,fm}(q)$ for total jet energies $Q = 34$ GeV and $Q = 91$ GeV.

The top panel shows $\sigma^{-1}\, d\sigma/dR_{cl}\ (fm^{-1})$ versus $R_{cl}\ (fm)$ for $q\bar{q}$-jet: $Q=100\ GeV$, with dashed curve $L_c=0.6\,fm$ ($B^{1/4}=230\ MeV$) and solid curve $L_c=0.8\,fm$ ($B^{1/4}=180\ MeV$).

The bottom panel shows $\sigma^{-1}\, d\sigma/dM_{cl}\ (GeV^{-1})$ versus $M_{cl}\ (GeV)$ for $q\bar{q}$-jet: $Q=100\ GeV$, with dashed curve $L_c=0.6\,fm$ ($B^{1/4}=230\ MeV$) and solid curve $L_c=0.8\,fm$ ($B^{1/4}=180\ MeV$).